Instance-Specific Production Control in Circular Manufacturing Using Gaussian Mixture Models

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Abstract. Circular manufacturing systems are characterized by high uncertainty, as the condition of each returned product is individual. This results in stochastic process behavior that directly affects production control decisions. Deterministic routing rules and fixed process parameters, which are common in linear production, are therefore insufficient. This paper proposes a probabilistic production control framework for circular factories based on Gaussian Mixture Models (GMMs). Products and their components are represented as probabilistic entities, with their process-related characteristics modeled using GMMs trained on historical shopfloor data, prior process steps for the specific instance, and inspection results. For each product and component instance, the models predict the process duration, the probability of successful execution, and the risk of failure or damage. These probabilistic predictions enable instance-specific decisions on process and station selection, as well as the adaptive choice between direct processing and intermediate inspection steps. By explicitly capturing multi-modal condition states and uncertainty, the framework enables risk-aware scheduling and routing decisions and can support robust and responsive control decisions compared to deterministic control approaches. The proposed method thus operationalizes data-driven uncertainty modeling for adaptive production control in circular manufacturing systems.

Keywords: Circular Manufacturing · Remanufacturing · Production Control

1 Introduction

The transition from linear production systems to circular manufacturing introduces new requirements for production planning and control [18]. In conventional manufacturing, incoming materials, product states, process sequences, and processing times can often be assumed to be sufficiently predictable. Circular manufacturing systems, by contrast, process returned products whose condition depends on individual usage histories, degradation mechanisms, prior maintenance,

and incomplete traceability information [6]. As a result, each returned instance may differ substantially from other instances of the same nominal product class [8].

This instance specific uncertainty directly affects circular value creation processes such as inspection, disassembly, reprocessing, and reassembly [14]. Depending on the condition of a returned product or component, the same operation may require different processing times, lead to different quality outcomes, or involve different risks of process failure and component damage. Therefore, production control decisions such as routing, scheduling, process selection, inspection, and resource allocation must increasingly account for the individual state and risk profile of each instance [3].

Conventional production control approaches are largely based on deterministic assumptions regarding process sequences and processing times [2]. While these assumptions may be suitable for highly standardized linear production environments, they become restrictive in circular manufacturing systems where process behavior is often stochastic and condition dependent [6]. Decisions based only on average parameter estimates or fixed routing logic can therefore lead to inefficient resource utilization and increased risk of process failure or component damage.

Recent advances in inspection, traceability, and shopfloor data acquisition provide increasing access to condition related information during circular value creation. At the same time, data driven methods create new opportunities to model uncertainty explicitly and use it as a decision relevant quantity[11]. However, existing approaches still provide limited support for connecting probabilistic representations of instance specific product states with operational production control decisions [15][7].

This paper addresses this gap by proposing a conceptual framework for probabilistic, instance specific production control in circular manufacturing. The framework represents returned products and their components as probabilistic entities whose process relevant characteristics are modeled using Gaussian Mixture Models. Based on information from prior process steps, inspection results, sensor observations, and historical shopfloor data, the probabilistic state of each instance can be updated during its passage through circular factories. These updated states provide decision relevant information for adaptive and risk aware production control.

The contribution of this paper is twofold. First, it introduces a probabilistic framework for representing the uncertain and dynamically evolving condition states of product and component instances in circular manufacturing. Second, it shows how this representation can be translated into operational production control decisions. The remainder of the paper is structured as follows. Section 2 introduces the control problem in circular production systems. Section 3 reviews related work. Section 4 presents the proposed GMM based conceptual framework. Section 5 discusses the integration into production control as well as expected potentials and implementation challenges. Section 6 concludes the paper and outlines future research directions.

2 Controlling Circular Production Systems

Circular production systems represent a specific production control problem with returned products and components entering the system in uncertain and instance specific conditions [12]. Even instances of the same nominal product type may require different process routes, resources, or inspection efforts. As a result, circular production cannot rely solely on fixed process plans and deterministic processing parameters.

From a control perspective, each product or component instance must be handled through a sequence of decisions during execution. The control system has to determine which process should be performed next, whether additional inspection is required, and which station should execute the operation [3]. These decisions are interdependent, since each action changes both the physical state of the instance and the information available for subsequent decisions.

This makes production control in circular factories a sequential decision problem under uncertainty. The controller has to balance processing efficiency, resource availability, process risk, and information acquisition. Direct processing may reduce lead time, but may increase the risk of unsuitable process or station assignments when the instance condition is uncertain. Additional inspection may improve decision quality, but consumes time and capacity.

Therefore, circular production control requires an instance specific information basis that evolves during execution. Such an information basis must support adaptive decisions by representing not only the expected process behavior of an instance, but also the uncertainty and risk associated with alternative actions.

3 Related Work

Related work on circular manufacturing and remanufacturing shows that planning and control become substantially more complex when returned products enter the system with uncertain quality, quantity, timing, and processing behavior. [17] identify uncertainty management as a central problem in remanufacturing, particularly due to changing of return flows and product quality. [9] review production planning and control for remanufacturing and show that existing approaches address several complicating characteristics, but that their integrated consideration remains limited.

A second stream of work addresses scheduling and control decisions under uncertainty. Existing studies consider uncertainty in disassembly, remanufacturing, repair, reassembly, and their integration. [1] propose integrated control policies for production, return replenishment, and inspection in hybrid manufacturing remanufacturing systems under stochastic and dynamic conditions. [19] considers failure probabilities for station and process selection; however, these probabilities are estimated at the class level rather than at the instance level. These contributions show that inspection and stochastic process behavior are already recognized as relevant decision variables. However, uncertainty is commonly represented at an aggregate, policy, or quality class level rather than as a continuously updated

probabilistic state of each individual product or component instance.
A third stream is related to inspection systems, digital twins, and control architectures for circular factories. [10] show that core inspection in remanufacturing is characterized by substantial uncertainty and that systematic automation of visual inspection remains challenging. Digital twin research emphasizes the relevance of lifecycle data, feedback loops, monitoring, prediction, and optimization for circular manufacturing decision support [16]. Recent architecture oriented work further proposes control structures for robust and resilient circular factories under uncertain conditions, including the integration of production stations, inspection stations, and intelligent intralogistics [3]. These approaches provide an important systems perspective, but they do not yet specify how uncertain product condition can be represented as an instance specific probabilistic state and directly translated into routing, inspection, and station selection decisions.
The remaining gap is therefore the missing link between probabilistic instance state representation and operational production control. Existing work recognizes uncertainty in circular manufacturing and addresses selected control decisions under stochastic conditions. However, there is still limited support for an explicit probabilistic state layer that represents the evolving condition of each returned instance and provides decision relevant quantities for adaptive and risk aware control.

4 GMM Based Comceptual Framework

The proposed framework provides a probabilistic information layer for instance specific production control in circular manufacturing systems. Its purpose is to represent the uncertain state of returned products and components, update this representation during processing, and derive control relevant quantities for subsequent production decisions. The framework therefore connects three elements: instance specific data, probabilistic state representation, and production control logic. Gaussian Mixture Models are used as the central modeling approach because they can represent heterogeneous and multimodal condition states without assigning each instance to one fixed quality class.

4.1 Instance Specific Data Representation

Each returned product or component instance is described by a set of available product, process, and inspection data. This data may include product type, return source, inspection results, sensor observations, observed processing times, and information from prior process steps. Since circular factories often process instances with incomplete and heterogeneous information, not all data sources are available for every instance. The framework therefore uses the available information to construct an instance specific feature representation that summarizes the current knowledge about the product or component.
This feature representation is not interpreted as a deterministic description of the actual condition. Instead, it serves as the basis for estimating a probabilistic state. For example, the same nominal component type may include lightly

worn, heavily worn, deformed, or partially damaged instances. These latent condition states are often not directly observable before processing. Therefore, the framework represents the condition of an instance as uncertain and updates this representation as additional information becomes available.

4.2 Probabilistic State Representation Using Gaussian Mixture Models

A Gaussian Mixture Model represents an uncertain quantity as a weighted sum of Gaussian distributions, each describing one typical condition group, with the weights indicating how likely the instance belongs to each group.

Let X_i denote the uncertain condition state of product or component instance i. The state may include process relevant attributes such as wear, corrosion, contamination, deformation, residual functionality, accessibility, or hidden damage. Instead of describing X_i by a single expected value or assigning it to a discrete quality class, the framework models it by a Gaussian Mixture Model which is a weighted sum of Gaussian densities:

$$\mathcal{G}_X(x|g) = \sum_{i=1}^{K} \pi_i \mathcal{N}_X(x|m_i, \Sigma_i), \tag{1}$$

where the parameter tuple is

$$g := (K, \{\pi_i, m_i, \Sigma_i\}_{i=1,\dots,K}), \tag{2}$$

with mixture weights $\pi_i \geq 0$, $\sum_{i=1}^{K} \pi_i = 1$, component means $m_i \in \mathbb{R}^d$, and positive definite covariance matrices $\Sigma_i \in \mathbb{R}^{d\times d}$ [5], [13].

Each mixture component can be interpreted as one latent condition or process behavior regime. The mixture weights describe how likely it is that the current instance belongs to these regimes. Figure 1 provides a graphical representation of such a mixture model, showing the individual Gaussian components, their weights, and the resulting overall density for the process duration of an instance.

This representation is suitable for circular manufacturing because returned products often originate from different usage contexts and degradation histories. As a result, the population of returned instances is usually heterogeneous and may contain several distinct condition groups. A GMM can capture such multimodal uncertainty while still providing a compact probabilistic representation that can be updated and evaluated during production control.

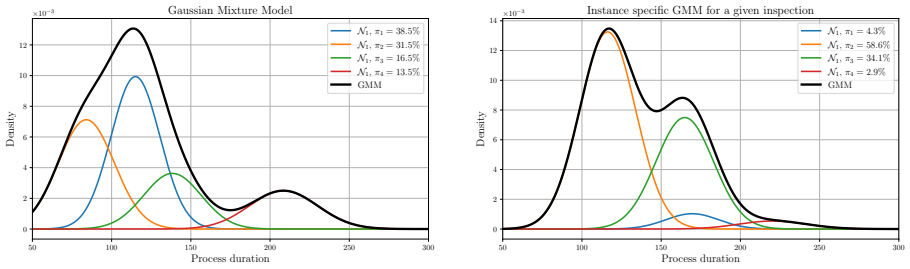
At the beginning of the process, the instance state can be initialized using prior information from historical shopfloor data, return histories, product type information, or engineering knowledge. This prior state represents the initial belief about the condition of the instance before detailed inspection or processing has been performed. During execution, this belief is refined by incorporating new observations.

4.3 Sequential Updating Through Inspection and Process Observations

A key element of the framework is that the probabilistic state of an instance is not static. Circular production processes often reveal information gradually. Initial inspection may provide first indications of wear or damage, disassembly may reveal hidden defects, cleaning may expose corrosion, and process observations such as force signals or realized processing times may provide additional evidence about the actual condition.

After each inspection or process step, the available observations are used to update the probabilistic state of the instance. Conceptually, the prior belief before an observation is transformed into a posterior belief after the observation (as demonstrated in figure 1). In this example, the inspection observation shifts the weights from four spread out condition regimes in Figure 1a towards two dominant regimes in Figure 1a which reduces the uncertainty of the predicted process duration. This posterior distribution then becomes the updated information state for the next control decision. In this way, the framework establishes a closed information loop between production execution, inspection, and probabilistic state estimation.

This sequential updating is particularly relevant for circular factories because the uncertainty of a returned product or component can often only be reduced during processing. The controller therefore does not rely on a fixed classification assigned at the beginning of the process. Instead, it continuously refines the instance specific state representation and adapts subsequent decisions to the updated uncertainty profile.



(a) Initial estimation process duration (b) Updated estimation process duration

Fig. 1: Update of an instance specific estimated process duration

4.4 Prediction of Process Relevant Quantities

The probabilistic state representation becomes useful for production control when it is translated into process relevant predictions. For a given product instance i, process alternative p, and station s, the framework estimates decision relevant quantities such as expected processing time, probability of process failure and probability of component damage: $E[T_{i,p,s}]$, $P(F_{i,p,s})$, $P(D_{i,p,s})$.These quantities are estimated for feasible process and station alternatives based on

the current posterior state of the instance. The use of full probability distributions instead of point estimates allows production control to distinguish between instances with similar expected processing times but different risk profiles. For example, two components may have the same expected refurbishment time, while one has a substantially higher probability of breakage due to uncertain damage. Such differences are highly relevant for process selection, station assignment, routing, and inspection decisions.

4.5 Interface to Production Control

The output of the GMM based framework serves as an input for adaptive production control. One possible way to connect the probabilistic predictions to a control decision is a risk aware score for each feasible combination of instance, process alternative, and station:

$$J(i,p,s) = \alpha\,\mathbb{E}[T_{i,p,s}] + \beta\,\mathbb{P}(F_{i,p,s}) + \gamma\,\mathbb{P}(D_{i,p,s}), \tag{3}$$

where α, β, and $\gamma \geq 0$ are weighting factors that reflect the relative importance of processing time, failure risk, and damage risk. The controller can then select the process and station combination with the lowest score among the feasible alternatives:

$$(p^*, s^*) = \arg\min_{(p,s)\in\mathcal{A}_i} J(i,p,s), \tag{4}$$

where $\mathcal{A}_i$ denotes the feasible action set for instance i.
This formulation is intentionally generic and can be extended by additional criteria such as queue length, due date urgency, energy consumption, resource capability, or inspection effort.
The risk aware score illustrates how probabilistic instance information can be operationalized for production control. The GMM itself does not replace the control logic. Rather, it provides the uncertainty aware input required to make more informed decisions. In addition to process and station selection, the same principle can support routing, prioritization, and the decision whether an additional inspection should be performed before committing an instance to a risky process. The following section describes this integration into production control in more detail.

5 Integration into Production Control

The proposed framework enables production control to move beyond nominal processing parameters and fixed routing logic. Process and station selection are no longer based only on expected processing time, but on the trade off between productivity and risk for the specific instance. For example, an instance with an elevated damage probability may be assigned to a more robust station or a less aggressive process variant, even if this increases processing time. Conversely,

instances with low uncertainty and low risk can be routed more directly through the system. In this way, products of the same nominal type are not treated as equivalent jobs, but are controlled according to their current condition and risk profile.

The resulting production control logic follows a closed loop. An instance enters the system with an initial probabilistic state. Based on this state, the control system evaluates the feasible actions and selects the next operation. After execution, new information becomes available, such as inspection results, realized processing times, observed defects, process failures, or damage events. This information is used to update the instance state, which then forms the basis for the next control decision. Estimation, decision, execution, and updating are therefore continuously linked during the passage of an instance through the circular production system.

The expected potential of this closed loop logic lies in a more robust and responsive handling of uncertain return flows. By considering instance specific uncertainty, the framework can support more suitable process assignments, more targeted use of inspection capacity, and better matching between product condition and resource capability. It can also reduce the risk of avoidable failures, rework, scrap, or component damage caused by unsuitable routing or station selection decisions.

At the same time, several implementation challenges remain. A first challenge is the availability and quality of instance level data. The approach requires informative historical process data, inspection results, and feedback from production execution. In many circular production systems, such data may be incomplete, noisy, or distributed across different information systems. A second challenge concerns model fitting and maintenance. The number of mixture components, the stability of learned condition regimes, and the updating of models over time have to be handled carefully to avoid overfitting or unstable predictions.

A further challenge is the computational effort required for operational control. Production decisions often have to be made under time constraints, especially in flexible production systems with many simultaneously active instances. Practical implementations therefore require efficient update mechanisms and reduced model complexity. Finally, the framework has to be integrated into existing shopfloor IT, such as MES, digital twins, scheduling systems, or dispatching logic. Since practitioners are often used to deterministic parameters and rule based control, the probabilistic outputs must also be interpretable and transparent.

The proposed integration should therefore be understood as an uncertainty aware decision layer rather than a complete replacement of existing production control methods. Its main function is to enrich established control mechanisms such as [4] with instance specific information about expected process behavior and associated risks. This enables circular production systems to move from deterministic execution logic toward adaptive control decisions that explicitly account for uncertainty, condition dependency, and progressive information availability.

6 Conclusion

This paper proposes a conceptual framework for probabilistic, instance specific production control in circular manufacturing systems. Circular factories process returned products whose condition is heterogeneous, uncertain, and only partially observable, which directly influences process duration, failure probability and damage risk. Deterministic production control based on fixed process plans and nominal parameters is therefore insufficient for many circular production environments.

To address this challenge, the framework represents product and component instances as probabilistic entities using Gaussian Mixture Models, capturing multimodal condition states and treating uncertainty as a decision relevant quantity. Based on return information, inspection results, process observations, and historical shopfloor data, the state of each instance is updated during execution and translated into control relevant quantities such as expected processing time, failure probability and damage probability. This enables process selection, station assignment and inspection decisions to account for the individual state and risk profile of each returned instance, so that uncertainty becomes an explicit input for adaptive and risk aware control rather than a mere disturbance.

Future work should focus on the operationalization and validation of the concept. This includes a simulation based comparison of GMM based control with deterministic and quality class based approaches, the development of data structures and interfaces for integration into existing shopfloor systems such as MES, digital twins and scheduling systems, efficient methods for model updating and mixture reduction to enable real time applicability, and industrial case studies to evaluate the practical benefits with respect to throughput, lead time, resource utilization, process robustness and recovery quality.

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